A, B, C, D, matrix[5]

$$U = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_{(2-2)} \\ 0 & 1 \end{bmatrix}. \tag{28}$$

The loss is given by

$$|R|^2 = \frac{(A+B+C+D)^2}{4}. \tag{29}$$

Substituting (25) into (29) yields

$$|R|^2 = \left[1 + \frac{m^2 G_T \dfrac{Q_{EC}}{Q_{OC}}}{2\left(G_T + \dfrac{Q_{EC}}{Q_{OC}}\right)} \right]^2 \tag{30}$$

but $L = 20 \log R$.

Substituting (30) in (31)

$$L = 20 \log \left[1 + \frac{m^2 G_T \dfrac{Q_{EC}}{Q_{OC}}}{2\left(G_T + \dfrac{Q_{EC}}{Q_{OC}}\right)} \right]. \tag{31}$$

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[5] E. A. Guillemin, "Communication Networks," John Wiley and Sons, Inc., New York, N. Y., vol. 2, ch. 4, pp. 132–180; 1947.

Cooling of Microwave Crystal Mixers and Antennas*

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Summary—**The development of low-noise mixer crystals has reached the point where the noise figure is approaching fundamental, theoretical limits. The desire for still greater sensitivity has led to the consideration of other possible means for noise reduction. This paper will discuss two possibilities: physically cooling the mixer crystal, and using an antenna directed toward background noise which is lower than room temperature. The improvement which can be realized increases rapidly as the room-temperature noise figure is reduced.**

RECEIVER SENSITIVITY IMPROVEMENT FROM "COLD ANTENNA"

THE IMPROVEMENT ratio of receiver sensitivity as the antenna looks at free space, to the sensitivity as it looks at a room temperature background, increases hyperbolically as the noise figure is improved.

$$\frac{(N_0)T_0}{(N_0)T_s} \doteq \frac{F}{F + \dfrac{T_s}{T_0} - 1} = [\mathrm{SR}]_A. \tag{1}$$

Here $(N_0)T_0$ is the output noise in the receiver as it looks at a room-temperature source, T_0, and $(N_0)T_s$ is the output noise in the receiver as it looks at the free-space background T_s. F represents the receiver noise figure. $[\mathrm{SR}]_A$ is the receiver sensitivity improvement ratio due to the cold antenna.

Because of the rapidity with which this ratio improves as F approaches its theoretical limit 1, it becomes economical to think of cooling the mixer *if* it has a very low F to begin with. X-band receivers using narrow-band techniques and low-noise germanium mixer crystals have noise figures low enough to make this improvement in sensitivity important and feasible.

RECEIVER SENSITIVITY IMPROVEMENT FROM COOLING MIXER

The noise figure of a receiver using a crystal is given by

$$F = L_x[t_x + F_{\mathrm{if}} - 1] \tag{2}$$

where F is the receiver noise figure, L_x is crystal conversion loss, t_x is crystal noise temperature, and F_{if} is the IF noise figure. This formula is not convenient for discussing improvements due to cooling because t_x is a function of L_x.[1] The functional relationship is

$$t_x = \begin{cases} \bar{t}\left(1 + \dfrac{1}{L_x}\right) + \dfrac{1}{L_x} & \text{narrowband} \\[2ex] \bar{t}\left(1 - \dfrac{2}{L_x}\right) + \dfrac{2}{L_x} & \text{broadband.} \end{cases} \tag{3}$$

Here $\bar{t}$ is the noise temperature due only to the crystal. Substituting back in (2)

$$F = [L_x\bar{t} - (1, 2)]\bar{t} + (1, 2) + L_x(F_{\mathrm{if}} - 1) \tag{4}$$

where the 1 and 2 pertain to narrowband and broadband, respectively. Now

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* Manuscript received by the PGMTT, June 14, 1956.

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[1] The harmonics of the local oscillator are assumed to be all shorted.

$$|\mathrm{SR}]_M = \frac{(L_x - 1,\, 2)\bar{t} + (1,\, 2) + L_x(F_{\mathrm{if}} - 1)}{(L_x - 1,\, 2)\bar{t}\dfrac{T}{T_0} + (1,\, 2) + L_x(F_{\mathrm{if}} - 1)} \tag{5}$$

where T is the mixer temperature and T_0 is room temperature and SR_M is the receiver-sensitivity-improvement-ratio due to cooling the mixer.

Now $\bar{t}$ is proportional to temperature if it is composed of any combination of shot and thermal noise. Theoretically, at least, it should decrease linearly with temperature. Cooling to 70°K without increasing L_x[2] should be practical. Therefore $\bar{t}$ can be cut by approximately a factor of four. Table I shows how the operation of a 1N263 diode could be improved by cooling and looking at a cold target area, under the assumption of an ideal F_{if} and of an F_{if} of 1.3 which is presently obtainable. Using selected 1N263 diodes the following room temperature values were measured:

TABLE I

IMPROVEMENT IN SENSITIVITY BY COOLING THE MIXER AND ANTENNA

		F at 300°K		SR by cooling Mixer to 70°K		Additional SR by Antenna Temp. of 10°K	
		Ratio	db	Ratio	db	Ratio	db
BB	$F_{\mathrm{if}}=1.3$	3.9	5.9	1.15	0.6	1.41	1.5
NB		3.5	5.4	1.60	2.0	1.84	2.6
BB	$F_{\mathrm{if}}=1.0$	3.0	4.7	1.20	0.8	1.67	2.2
NB		2.75	4.4	1.90	2.8	3.2	5.0

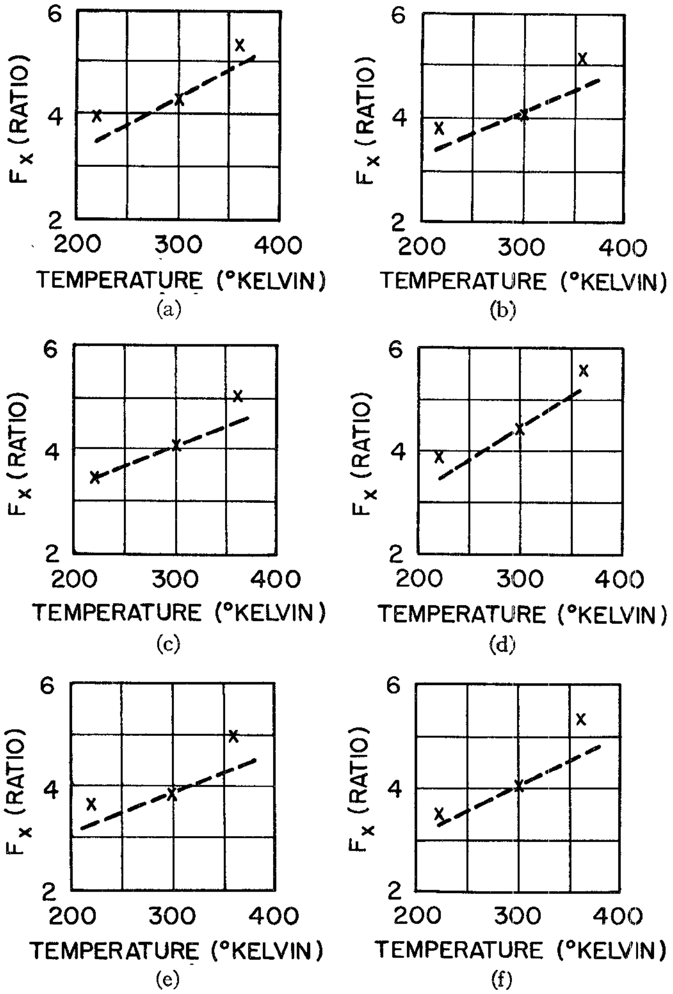
$$L_x(BB) = 3.0 \qquad t_x(BB) = 1.0$$

$$L_x(NB) = 2.5 \qquad t_x(NB) = 1.1.$$

For the obtainable broad-band mixer with a 1.3 F_{if} and a 5.9 db room-temperature noise figure, an improvement of 0.6 db comes from cooling the mixer and another improvement of 1.5 db by looking at a background temperature of 10°K. The cumulative effect is a 2.1 db improvement. The equivalent total improvement for the obtainable narrow-band mixer would be 4.6 db. If the F_{if} could be cut to 1.0, the total improvement could be increased to 7.8 db, for a narrow-band mixer.

EXPERIMENTAL RESULTS

The results of noise-figure measurements on an experimental broad-band X-band receiver using germanium crystals are plotted as a function of absolute temperature in Fig. 1. The theoretical curves are computed using the room-temperature noise figure as a basis. The crystals were not rematched at either the high or low temperature which may explain why the discrepancies are all on the high side.



(a) (b) (c) (d) (e) (f)

Fig. 1—Noise figure vs temperature. This figure shows the comparison between theoretical and experimental noise figures obtained for six 1N263 mixer crystals. The crystals were matched at room temperature but were not rematched at the other temperatures. Failure to rematch may account for the discrepancies which are all on the high side.

Experiments at liquid-air temperatures are not feasible at the present time because the entire receiver front end would require redesign in order to permit operation at these temperatures. The crystal package would also require redesign with operation at low temperature in mind.

CONCLUSION

The nature of both (1) and (5) is such that the amount of improvement possible increases greatly as the room-temperature noise figure is reduced. In receivers where sensitivity is of paramount importance the low-noise figures already attainable in narrow-band receivers using low-noise germanium crystals makes further refinement, such as cooling the mixer, practical and worthwhile.

In the present narrow-band mixer the cumulative improvement in sensitivity of cooling the mixer and of looking at a cold background could be as much as 4.6 db.

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[2] The resistivity of the germanium would increase slightly and the log slope of the iv characteristic would increase. The net effect on L_x would be small.